Received: *June 15, 2001*, Accepted: *July 26, 2001*

Proposal for background independent Berkovits'superstring field theory

Josef Klusoň

Department of Theoretical Physics and Astrophysics
Faculty of Science, Masaryk University
Kotlářská 2, 611 37, Brno, Czech Republic
E-mail: klu@physics.muni.cz

Abstract: In this paper we would like to propose the background independent formulation of Berkovits' superstring field theory. Then we will show that the solution of equation of motion of this theory leads to the Berkovits' superstring field theory formulated around particular CFT background.

Keywords: Superstrings and Heterotic Strings, D-branes.

JHEP07(2001)039

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1. Introduction

It was shown in [28] that the fundamental formulation of bosonic open string field theory [2] can be formulated as a pure cubic string field theory action. This action does not contain any BRST operator and it is formally background independent. When we then expand string field around any solution of the equation of motion arising from this pure cubic string field action we obtain exactly Witten's bosonic field theory with the BRST operator constructed from the field obeying the equation of motion of the pure cubic string field theory. This approach has been further developed in [29], where another solutions of the pure cubic string field theory that do not correspond to any usual BRST operator were found.

Recently there was a great interest in the formulation of the open bosonic string field theory around the closed string vacuum (Vacuum string field theory-VSFT) [11]–[14], [16]–[19], [21, 22, 23, 33, 34] that arises as a final point of the tachyon condensation on D25-brane (For a very nice review of the string field theory and its relation to the problem of the tachyon condensation, see [1].) VSFT is characterised by BRST operator with trivial cohomology so that there are not any physical open string excitations. This BRST operator is constructed from ghost fields only so that vacuum string field theory is formally background independent. VSFT seems to be a very promising area of research which could lead to better understanding of the string theory.

The problem of the tachyon condensation was studied in superstring theory as well. It seems that the most promising approach to the study of the tachyon condensation in this context is based on Berkovits' superstring field theory[1] [5, 6]. Two major advantages of Berkovits superstring field theory are that it is manifestly SO(3, 1)

[1] For another interesting formulation of superstring field theory, see [7, 8, 9].

super-Poincaré invariant and that it does not require contact terms to remove tree-level divergences [25] (For recent review of Berkovits superstring field theory (NSFT), see [26].) In particular, it was shown that the calculation of the tachyon potential in NSFT is in a very good agreement with Sen's conjecture [27]. The tachyon kink and lump solution was also analysed (For recent work, see [15], for list of references, see [1].) However this theory is formulated for Neveu-Schwarz open string sector only because it is not known how to include Ramond sector in manifestly Lorentz invariant manner.

In this paper we would like to ask the question whether there could be such a formulation of string field theory from which the NSFT arises in a natural way as the Witten's open string field theory emerges from the pure cubic string field action. Since pure cubic string field theory does not contain any BRST operator it is formally background independent, we can ask the question whether there is such a formulation of the NSFT theory that is background independent as well. In other words, we are looking for a string field theory action that does not depend on any particular matter conformal field theory background. It seems to us that the second formulation of the problem is the appropriate one for Berkovits' superstring field theory. This can be seen as follows. In Witten's string field theory the fundamental object is a ghost number one string field. Since the BRST operator carries the same ghost number there is no problem to define BRST operator using string field as has been done in [28]. On the other hand, fundamental string field in NSFT is a string field of the ghost number and the picture number zero. At first sight it seems to be impossible to define string field theory action in background independent way. However recent results [11, 17, 18, 21] suggest that there can exist BRST operator that is constructed purely from the ghost field which is universal for any conformal field theory background. In fact, in a remarkable paper [30] the BRST operator for $N=1$ NSR string theory was given that resembles striking similarity with the terms presented in NSFT action.[2] It is then natural to presume that the background independent formulation of the NSFT theory could be based on the BRST operator Q_0 defined in [30] and explicit form of which will be given below.[3] Then using recent result [32] it is possible to find such a string field that is solution of the equation of motion of the background independent NSFT and that leads to the emergence of the correct BRST operator for any CFT background.

The plan of the paper is follows. In the next section 2 we briefly review NSFT theory defined on the BPS D-brane.

In section 3 we review the formulation of the open bosonic string field theory based on the pure cubic action [28, 29]. We will show how Witten's open bosonic string field theory action naturally emerges from the pure cubic string field action.

[2]We are very thankful to N. Berkovits for emphasising this form of the BRST operator and pointing out the reference [30].

[3]Similar suggestion has been proposed in [31].

We use these elegant calculations in section 4 where we propose a background independent formulation of the Berkovits string field theory. Then we will show that from this action we can obtain any NSFT action for any CFT background.

In conclusion 5 we will outline our results and suggest further extension of this work.

2. Review of superstring field theory

In this section we would like to review basic facts about Berkovits superstring field theory, for more details, see [1, 4, 5, 10, 26]. The general off-shell string field configuration in the[4] GSO(+) NS sector corresponding Grassmann even open string vertex operator Φ of ghost number 0 and picture number 0 in the combined conformal field theory of a $c = 15$ superconformal matter system, and b, c, β, γ ghost system with $c = -15$. We can also express β, γ in terms of ghost fields ξ, η, ϕ

$$\beta = e^{-\phi}\partial\xi\,, \qquad \gamma = \eta e^{\phi}\,, \tag{2.1}$$

the ghost number n_g and the picture number n_p assignments are as follows

$$\begin{aligned} b:\ n_g = -1,\ n_p = 0 \quad & c:\ n_g = 1,\ n_p = 0\,; \\ & e^{q\phi}:\ n_g = 0,\ n_p = q\,; \\ \xi:\ n_q = -1,\ n_p = 1 \quad & \eta:\ n_q = 1,\ n_p = -1\,. \end{aligned}$$

The BRST operator Q_B is given

$$Q_B = \oint dz j(z) = \int dz \left\{ c(T_m + T_{\xi\eta} + T_\phi) + c\partial c b + \eta e^{\phi} G_m - \eta\partial\eta e^{2\phi} b \right\}, \tag{2.2}$$

where

$$T_{\xi\eta} = \partial\xi\eta\,, \qquad T_\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi\,, \tag{2.3}$$

T_m is a matter stress tensor and G_m is a matter superconformal generator. Throughout this paper we will be working in units $\alpha' = 1$.

The string field action is given [4, 5]

$$\begin{aligned} S = \frac{1}{2}\int \Bigg(& (e^{-\Phi} \star Q_B e^{\Phi})(e^{-\Phi} \star \eta_0 e^{\Phi}) - \\ & - \int_0^1 dt (e^{-t\Phi} \star \partial_t e^{t\Phi}) \star \left\{ (e^{-t\Phi} \star Q_B e^{t\Phi}), (e^{-t\Phi} \star \eta_0 e^{t\Phi}) \right\} \Bigg)\,, \end{aligned} \tag{2.4}$$

where $\{A, B\} = A \star B + B \star A$ and $e^{-t\Phi} \star \partial_t e^{t\Phi} = \Phi$. Here the products and integral are defined by Witten's gluing prescription of the string. The exponential of string

[4] In this paper we will consider NSFT for BPS D-branes only.

field is defined in the same manner $e^{\Phi} = 1 + \Phi + \frac{1}{2}\Phi \star \Phi + \ldots$. The basis properties of Q_B, η_0 which we will need in our analysis (for more details, see [1] and reference therein) are

$$\begin{aligned}
Q_B^2 &= 0\,, \quad \eta_0^2 = 0\,, \quad \{Q_B, \eta_0\} = 0\,, \\
Q_B(\Phi_1 \star \Phi_2) &= Q_B(\Phi_1) \star \Phi_2 + \Phi_1 \star Q_B(\Phi_2)\,, \\
\eta_0(\Phi_1 \star \Phi_2) &= \eta_0(\Phi_1) \star \Phi_2 + \Phi_1 \star \eta_0(\Phi_2)\,, \\
\int Q_B(\ldots) &= 0\,, \quad \int \eta_0(\ldots) = 0\,,
\end{aligned} \tag{2.5}$$

where Φ_1, Φ_2 are Grassmann even fields.

For our purposes it will be useful to write the BRST operator in the form [30]

$$Q_B = e^{-R}\left(\frac{1}{2\pi i}\oint dz\gamma^2 b\right)e^{R}\,, \tag{2.6}$$

where

$$R = \frac{1}{2\pi i}\oint dz[cG_m e^{-\phi}e^{\chi} - \frac{1}{4}\partial(e^{-2\phi})e^{2\chi}c\partial c] = \frac{1}{2\pi i}\oint dz r(z) \tag{2.7}$$

and

$$\begin{aligned}
&\beta = e^{-\phi}\partial\xi\,, \quad \gamma = \eta e^{\phi}\,, \quad \xi(z)\eta(w) = \frac{1}{z-w}\,, \quad \phi(z)\phi(w) = -\log(z-w)\,, \\
&\xi = e^{\chi}\,, \quad \eta = e^{-\chi}\,, \quad \chi(z)\chi(w) = -\log(z-w)\,.
\end{aligned}$$

It is easy to see that

$$Q_0^2 = \frac{1}{2}\{Q_0, Q_0\} = 0\ , Q_0 = \frac{1}{2\pi i}\oint \gamma^2 b \tag{2.8}$$

and also

$$\{\eta_0, Q_0\} = 0\,. \tag{2.9}$$

Then we immediately obtain

$$Q_B^2 = e^{-R}Q_0^2 e^{R} = 0\,. \tag{2.10}$$

In other words, Q_B is a nilpotent operator. It was also shown in [30] that Q_B given in (2.6) anticommutes with η_0 in critical dimensions $D = 10$. From the fact that Q_0 is constructed from the ghost fields only it is the same for any CFT background so that it seems to be a natural BRST operator for a background independent formulation of NSFT in the similar way as in [11, 17, 18, 21]. On the other hand, it must be stressed that this operator has not trivial cohomology[5] so that the background independent formulation of NSFT should not be confused with the vacuum string field theory that describes the bosonic open string field theory after the tachyon condensation.

[5] We would like to thank N. Berkovits for stressing this point to us.

We mean that this seems to be natural fact since we are looking for a background independent formulation of the BPS D-brane so that there is not any tachyon field present and hence the process of the tachyon condensation cannot occur.

In the next section we briefly review the approach given in [28]. In particular, we will see how Witten's open bosonic string field theory naturally emerges from the pure cubic string field theory action.

3. Pure cubic action for string field theory

It was proposed in [28] that pregeometrical, background independent (at least formally) string field action has a form

$$S = \frac{1}{3}\int \Phi \star \Phi \star \Phi \,, \tag{3.1}$$

where Φ is Grassmann odd string field of ghost number one. The classical equation of motion is

$$\Phi \star \Phi = 0 \,. \tag{3.2}$$

As was shown in [28] when we expand around the classical solution

$$\Phi = \Phi_0 + \phi \tag{3.3}$$

we get an action for fluctuation

$$S = \int \frac{1}{2}\phi \star D_{\Phi_0}\phi + \frac{1}{3}\phi \star \phi \star \phi \,, \tag{3.4}$$

where

$$D_{\Phi_0} X = \Phi_0 \star X - (-1)^X X \star \Phi_0 \,. \tag{3.5}$$

It was shown in [28] that D_{Φ_0} is a derivation. In order to recovery Witten's form of the string field action [2] we must find field Φ_0 solving equation of motion and

$$D_{\Phi_0} = Q_B \,, \tag{3.6}$$

where Q_B is the BRST operator associated with some background. It was shown in [28] that such a field Φ_0 is uniquely given by the relation

$$\Phi_0 = Q^L \mathcal{I} \,, \tag{3.7}$$

where Q^L (Q^R) is the BRST charge density integrated over the left (right) half of the string ($Q = Q^R + Q^L$) and $\mathcal{I}$ is the identity operator of the algebra obeying

$$\mathcal{I} \star B = B \star \mathcal{I} = B \,, \qquad \forall B \,. \tag{3.8}$$

In Witten's open string field theory, Q_B represents a reference background and ϕ represents the second quantized fluctuation field around that background. As was shown in [28], shifting ϕ it is possible to eliminate this specific reference to a background. In their second-quantized formulation the backgrounds arise as solutions to the equations of motion.

It is natural to ask the question whether similar formalism works in case of NSFT as well. Although we will not be able to find an analogue of the pure cubic string field action for NSFT, we will see that it is possible (at least formally) to formulate the background independent version of NSFT based on the BRST operator constructed from the ghost field only.

4. Proposal for background independent NSFT

Since we will not perform any explicit calculation we will again use abstract Witten's formalism in string field theory [2]. We would like to find a formulation of the NSFT theory from which a general action (2.4) emerges in natural way as in [28, 29], or equivalently, we would like to find a formulation of the NSFT theory that does not depend on any particular CFT background as in case of VSFT. In fact, the second requirement is the more appropriate one for NSFT which can be seen from the following argument. In bosonic string field theory the fundamental object is a string field of the ghost number one so that it is natural that the BRST operator of the same ghost number can emerge from the pure cubic action. On the other hand in NSFT theory the fundamental field is a Grassmann ghost zero field so that it does not seem to be possible construct BRST operator from the action containing fundamental fields only without presence of any ghost number one operator. However, there is certainly fundamental BRST operator that does not depend on any particular background, which is the operator given in [30]

$$Q_0 = \frac{1}{2\pi i}\oint dz\gamma^2(z)b(z)\,. \tag{4.1}$$

It is clear that this operator does not depend on any background CFT so it is natural to propose background independent formulation of NSFT theory using this nilpotent operator

$$\begin{aligned}S = \frac{1}{2}\int\Big(&(e^{-\Phi}\star Q_0 e^{\Phi})(e^{-\Phi}\star\eta_0 e^{\Phi}) - \\ &-\int_0^1 dt(e^{-t\Phi}\star\partial_t e^{t\Phi})\star\left\{(e^{-t\Phi}\star Q_0 e^{t\Phi}),(e^{-t\Phi}\star\eta_0 e^{t\Phi})\right\}\Big).\end{aligned} \tag{4.2}$$

We would like to show, following recent analysis [32] that it is possible to find such a solution of the equation of motion arising from the previous action

$$\eta_0(e^{-\Phi_0}\star Q_0(e^{\Phi_0})) = 0 \tag{4.3}$$

that leads to the NSFT action with appropriate BRST operator Q_B.

Let us consider any string field Φ_0, corresponding to $G_0 = e^{\Phi_0}$, which is a solution of the equation of motion (4.3). In order to find a new form of BRST operator, we must study the behaviour of the fluctuation field around this solution [32]. For that reason we write general string field containing fluctuation around this solution as

$$G = G_0 \star h \,, \qquad h = e^{\phi} \,, \qquad G^{-1} = h^{-1} \star G_0^{-1} \,. \tag{4.4}$$

To see that this field really describes fluctuations around solution G_0 note that for $\phi = 0, G = G_0$. It is also clear that any string field in the form $e^{\Phi_0 + \phi'}$ can be always rewritten in the form given above.

Inserting this upper expression in (4.2) we obtain an action for ϕ. As was argued in [32] in order to find a new BRST operator we must ask the question what form of the equation of motion obeys shifted field $h = e^{\phi}$. Then it was shown that the new BRST operator has a form

$$Q_B(X) = Q_0(X) + A \star X - (-1)^X X \star A \,, A = G_0^{-1} \star Q_0(G_0) \tag{4.5}$$

and the string field action for the fluctuation field has exactly the same form as (2.4) with the BRST operator (4.5).

From (4.5) we obtain

$$(Q_B - Q_0)X = A \star X - (-1)^X X \star A \,. \tag{4.6}$$

Using results given in [28]

$$\begin{aligned} Q^R \mathcal{I} &= -Q^L \mathcal{I} \,, \quad Q = Q^R + Q^L \,, \quad \mathcal{I} \star X = X \star \mathcal{I} = X \,, \quad \forall X \,, \\ (Q^R X) \star Y &= -(-1)^X X \star Q^L(Y) \,, \quad \forall X, Y \,, \\ \{Q, Q^L\} &= 0 \,, \end{aligned}$$

we see that we can write A as

$$A = (Q_B - Q_0)^L \mathcal{I} \tag{4.7}$$

since then

$$\begin{aligned} (Q_B - Q_0)^L(\mathcal{I}) \star X &= -(Q_B - Q_0)^R(\mathcal{I}) \star X = \mathcal{I} \star (Q_B - Q_0)^L(X) \,, \\ -(-1)^X X \star (Q_B - Q_0)^L \mathcal{I} &= (Q_B - Q_0)^R(X) \star \mathcal{I} \,, \end{aligned}$$

after application of (4.7). Then we have

$$A \star X - (-1)^X X \star A = (Q_B - Q_0)^L(X) + (Q_B - Q_0)^R(X) = (Q_B - Q_0)(X) \,. \tag{4.8}$$

Since we know that Q_B, Q_0 are correct BRST operators that anticommute with η_0 [30], we can easily prove that A given in (4.7) solves the equation of motion (4.3).

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This can be seen as follows. The fact that $\{\eta_0, G_0\} = \{\eta_0, Q_B\}$ means that when we express these operators using appropriate currents then OPE between $\eta(z)$ and $j_0(z), j_B(z)$ is non-singular

$$\eta(z)j_0(w) \sim O(0)\,, \qquad \eta(z)j_B(w) \sim O(0)\,, \tag{4.9}$$

If we do a contour integral over z in upper expressions we obtain the operator η_0 and next integration of $j_{0,B}(w)$ over left half of the string we find that the anticommutator is equal to zero

$$\{\eta_0, Q_0^L\} = 0\,, \qquad \{\eta_0, Q_B^L\} = 0\,. \tag{4.10}$$

Consequently we get from (4.7)

$$\eta_0(A) = \eta_0(Q_B - Q_0)^L \mathcal{I} = -(Q_B - Q_0)^L(\eta_0(\mathcal{I})) = 0\,, \tag{4.11}$$

where we have used

$$\eta_0(X) = \eta_0(X \star \mathcal{I}) = \eta_0(X) \star \mathcal{I} + (-1)^X X \star \eta_0(\mathcal{I}) \Rightarrow \eta_0(\mathcal{I}) = 0\,. \tag{4.12}$$

From (4.11) we see that A given in (4.7) solves the equation of motion (4.3). Now we explicitly determine Φ_0 in $A = e^{-\Phi_0}Q_0(e^{\Phi_0})$.

Let us define the function

$$F(t) = e^{-t\Phi_0}Q_0(e^{t\Phi_0})\,. \tag{4.13}$$

We can make an expansion around the point $t = 0$ where $F(0) = Q_0(1) = 0$

$$F(t) = \sum_{n=0}^{\infty} \frac{1}{n!} \left.\frac{d^n F(t)}{dt^n}\right|_{t=0} t^n\,. \tag{4.14}$$

The first derivative is equal to

$$\frac{dF}{dt} = -e^{-t\Phi_0}\Phi_0 Q_0(e^{t\Phi_0}) + e^{-t\Phi_0}Q_0(\Phi_0 e^{t\Phi_0}) = e^{-t\Phi_0}Q_0(\Phi_0)e^{t\Phi_0} \tag{4.15}$$

and the second one

$$\frac{d^2F(t)}{dt^2} = -e^{-t\Phi_0}\Phi_0 Q_0(\Phi_0)e^{t\Phi_0} + e^{-t\Phi_0}Q_0(\Phi_0)\Phi_0 e^{t\Phi_0} = e^{-t\Phi_0}[Q_0(\Phi_0), \Phi_0]e^{t\Phi_0}\,. \tag{4.16}$$

Generally we have

$$\frac{d^nF(t)}{dt^n} = e^{-t\Phi_0}\overbrace{[[Q_0(\Phi_0), \Phi_0], \ldots], \Phi_0]}^{n-1} e^{t\Phi_0}\,, \qquad n > 1 \tag{4.17}$$

and consequently

$$F(t = 1) = A = e^{-\Phi_0}Q_0(e^{\Phi_0}) = Q_0(\Phi_0) + \sum_{n=2}^{\infty} \frac{1}{n!} \overbrace{[[Q_0(\Phi_0), \Phi_0], \ldots,]\Phi_0]}^{n-1}\,. \tag{4.18}$$

We would like to compare this expression with the expression for BRST operator [30] given in (2.6) and with Q_0 given in (2.8).

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As in calculation performed above we define

$$F(t)=e^{-Rt}Q_0e^{Rt}=F(0)+\sum_{n=1}^{\infty}\frac{1}{n!}\frac{d^nF(t)}{dt^n}t^n \tag{4.19}$$

then

$$\begin{aligned}
F(0)&=Q_0\,, \quad \frac{dF}{dt}=-e^{-tR}RQ_0e^{tR}+e^{-tR}Q_0Re^{tR}=e^{-tR}[Q_0,R]e^{tR}\,,\\
\frac{d^2F}{dt^2}&=-e^{-tR}R[Q_0,R]e^{tR}+e^{-tR}[Q_0,R]Re^{Rt}=e^{-tR}[[Q_0,R],R]]e^{tR}\,,\\
\frac{d^nF}{dt^n}&=e^{-tR}\overbrace{[[Q_0,R],\dots,],R]}^{n}e^{tR}
\end{aligned} \tag{4.20}$$

and consequently

$$Q_B=F(1)=Q_0+\sum_{n=1}^{\infty}\frac{1}{n!}\overbrace{[[Q_0,R],R],\dots]R]}^{n}\,. \tag{4.21}$$

Then we obtain from (4.7)

$$Q_0(\Phi_0)+\sum_{n=2}^{\infty}\frac{1}{n!}\overbrace{[[Q_0(\Phi_0),\Phi_0],\dots,]\Phi_0]}^{n-1}=\left(\sum_{n=1}^{\infty}\frac{1}{n!}\overbrace{[[Q_B,R],\dots,],R]}^{n}\right)^L\mathcal{I}\,. \tag{4.22}$$

Now we must explain more carefully what we mean by the symbols $\mathcal{X}^{L,R}$ for any operator $\mathcal{X}$. Firstly, let us presume that the world-sheet operator $\mathcal{X}$ is defined by [6]

$$\mathcal{X}=\frac{1}{2\pi i}\oint_C dzx(z)\,, \tag{4.23}$$

where C is any closed curve encircling the origin of the conformal plane. It is clear that the operator given above can be written as a sum of two operators defined as integrals of world-sheet densities over left half or right half of the string, in particular for Q,R we get

$$Q_0=Q_0^R+Q_0^L\,, \qquad R=R^L+R^R\,, \tag{4.24}$$

where indices R,L correspond to the left, right half of the open string respectively. For example, Q_L can be defined as a contour integral of the holomorphic density over curve C_L that lies in the right half of the complex plane and Q_R as a contour integral over curve C_R that lies in the left half of the complex plane

$$Q_0^{R,L}=\frac{1}{2\pi i}\int_{C^R,C^L}dzj(z)_0\,, \qquad R^{R,L}=\frac{1}{2\pi i}\int_{C^R,C^L}dzr(z)\,. \tag{4.25}$$

[6] It is convenient to use "double trick" for open strings. We trade the holomorphic and antiholomorphic components of any field, defined in upper half plane, for a single holomorphic field defined in the whole complex plane.

From comments given above we immediately obtain

$$[Q_0^R, R^L] = 0 \tag{4.26}$$

since curves C_R, C_L have not common points and hence OPE between holomorphic densities is non-singular. Then for any operator $\mathcal{O}$ that is commutator of two operators $\mathcal{X}, \mathcal{Y}$ we get

$$\mathcal{O} = \mathcal{O}^R + \mathcal{O}^L = [\mathcal{X}, \mathcal{Y}] = [\mathcal{X}^L, \mathcal{Y}^L] + [\mathcal{X}^R, \mathcal{Y}^R] \tag{4.27}$$

and consequently

$$\mathcal{O}^L = [\mathcal{X}^L, \mathcal{Y}^L]\,, \qquad \mathcal{O}^R = [\mathcal{X}^R, \mathcal{Y}^R]\,. \tag{4.28}$$

Then we can write

$$[Q_0, R]^L \mathcal{I} = [Q_0^L, R^L]\mathcal{I}\,. \tag{4.29}$$

Using previous results we claim that the solution of the equation (4.22) has a form

$$\Phi_0 = R^L \mathcal{I}\,. \tag{4.30}$$

We will show that this expression really leads to the BRST operator Q_B. Firstly it is easy to see that (4.30) solves the following equation

$$Q_0(\Phi_0) = [Q_0^L, R^L]\mathcal{I} \tag{4.31}$$

since the left hand side of (4.31) is equal to

$$\begin{aligned} Q_0(\Phi_0) &= (Q_0^R + Q_0^L)(R^L \mathcal{I}) = Q_0^R R^L \mathcal{I} + Q_0^L R^L \mathcal{I} = R^L Q_0^R \mathcal{I} + Q_0^L R^L \mathcal{I} \\ &= -R^L Q_0^L \mathcal{I} + Q_0^L R^L \mathcal{I} = [Q_0^L, R^L]\mathcal{I} \end{aligned}$$

using (4.26) and (4.7). The second term in (4.22) is

$$[Q_0(\Phi_0), \Phi_0] = [[Q_0, R], R]^L \mathcal{I} = [[Q_0, R]^L, R^L]\mathcal{I} = [[Q_0^L, R^L], R^L]\mathcal{I}\,. \tag{4.32}$$

In order to see that (4.30) really solves upper expression and generally the whole equation (4.22) we must perform lot of calculations in the similar way as in [28]. Firstly, as in case of Q_0 we can prove

$$R(X) = R(X \star \mathcal{I}) = R(X) \star \mathcal{I} + X \star R(\mathcal{I}) \Rightarrow R(\mathcal{I}) = 0\,. \tag{4.33}$$

This result follows from the contour integration argument. Previous result implies

$$R^L \mathcal{I} = -R^R \mathcal{I}\,. \tag{4.34}$$

In the same way as in (4.7) we can write

$$R^R(X) \star Y = -X \star R^L(Y)\,. \tag{4.35}$$

Note that there is not a factor $(-1)^X$ since R is Grassmann even operator. Then we have

$$R^L\mathcal{I} \star R^L\mathcal{I} = -R^R\mathcal{I} \star R^L\mathcal{I} = \mathcal{I} \star R^L R^L(\mathcal{I}) = (R^L)^2(\mathcal{I})\,, \tag{4.36}$$

where we have used in the first step (4.34) and in the second step (4.35) in the form

$$R^R(\mathcal{I}) \star R^L(\mathcal{I}) = -\mathcal{I} \star R^L(R^L(\mathcal{I}))\,. \tag{4.37}$$

We also have

$$\begin{aligned} Q_0^R(R^L\mathcal{I}) \star \mathcal{I} &= -R^L\mathcal{I} \star Q_0^L\mathcal{I} \Rightarrow R^L Q_0^R(\mathcal{I}) \star \mathcal{I} = -R^L\mathcal{I} \star Q_0^L(\mathcal{I}) \Rightarrow \\ \Rightarrow R^L Q_0^L(\mathcal{I}) &= R^L(\mathcal{I}) \star Q_0^L(\mathcal{I})\,, \end{aligned} \tag{4.38}$$

where we have firstly used (4.26) and then (4.7). In the same way we can show that

$$\begin{aligned} R^R(Q_0^L\mathcal{I}) \star \mathcal{I} &= -Q_0^L(\mathcal{I}) \star R^L\mathcal{I} \Rightarrow Q_0^L R^R\mathcal{I} = -Q_0^L\mathcal{I} \star R^L\mathcal{I} \Rightarrow \\ &\Rightarrow Q_0^L(R^L\mathcal{I}) = Q_0^L\mathcal{I} \star R^L\mathcal{I}\,, \\ -Q_0^R((R^L)^2\mathcal{I}) \star \mathcal{I} &= (R^L)^2\mathcal{I} \star Q_0^L\mathcal{I} \Rightarrow (R^L)^2 Q_0^L\mathcal{I} = (R^L)^2\mathcal{I} \star Q^L\mathcal{I}\,, \\ R^R(Q_0^L(R^L\mathcal{I})) \star \mathcal{I} &= -Q_0^L(R^L\mathcal{I}) \star R^L\mathcal{I} \Rightarrow \\ \Rightarrow Q_0^L((R^L)^2\mathcal{I}) &= Q_0^L(R^L\mathcal{I}) \star R^L\mathcal{I} \Rightarrow Q_0^L(R^L)^2\mathcal{I} = Q_0^L\mathcal{I} \star (R^L)^2\mathcal{I}\,. \end{aligned}$$

Generally we obtain

$$\begin{aligned} (R^L)^n Q_0^L\mathcal{I} &= (R^L)^n\mathcal{I} \star Q_0^L\mathcal{I}\,, \\ Q_0^L(R^L)^n\mathcal{I} &= Q_0^L\mathcal{I} \star (R^L)^n\mathcal{I}\,. \end{aligned}$$

We can also write

$$\begin{aligned} (R^L)^n Q_0^L\mathcal{I} \star R^L\mathcal{I} &= -R^R((R^L)^n Q_0^L\mathcal{I}) \Rightarrow \\ &\Rightarrow (R^L)^n Q_0^L\mathcal{I} \star R^L\mathcal{I} = -(R^L)^n Q_0^L R^R\mathcal{I} \\ &\Rightarrow (R^L)^n Q_0^L\mathcal{I} \star R^L\mathcal{I} = (R^L)^n Q_0^L R^L\mathcal{I}\,; \\ R^L\mathcal{I} \star Q_0^L(R^L)^n\mathcal{I} &= -R^R\mathcal{I} \star Q_0^n(R^L)^n\mathcal{I} = \mathcal{I} \star R^L(Q_0^L(R^L)^n\mathcal{I}) = R^L Q_0^L(R^L)^n\mathcal{I}\,. \end{aligned}$$

Generally we have

$$\begin{aligned} R^L\mathcal{I} \star (R^L)^n Q_0^L(R^L)^m\mathcal{I} &= (R^L)^{n+1} Q_0^L(R^L)^m\mathcal{I}\,; \\ (R^L)^n Q_0^L(R^L)^m\mathcal{I} \star R^L\mathcal{I} &= (R^L)^n Q_0^L(R^L)^{m+1}\mathcal{I}\,. \end{aligned}$$

Using these results we can easily calculate

$$\begin{aligned} [Q_0(\Phi_0), \Phi_0] &= [[Q_0^L, R^L]\mathcal{I}, R^L\mathcal{I}] \\ &= Q_0^L\mathcal{I} \star R^L\mathcal{I} \star R^L\mathcal{I} - R^L\mathcal{I} \star Q_0^L R^L\mathcal{I} - \\ &\quad -R^L\mathcal{I} \star Q_0^L R^L\mathcal{I} + R^L\mathcal{I} \star R^L\mathcal{I} \star Q_0^L\mathcal{I} \\ &= (Q_0^L(R^L)^2 - R^L Q_0^L R^L - R^L Q_0^L R^L + (R^L)^2 Q_0^L)\mathcal{I} = [[Q_0^L, R^L], R^L]\mathcal{I} \end{aligned}$$

which proves (4.32). Generally we have a result

$$\overbrace{[[Q_0(\Phi_0),\Phi_0],\ldots],\Phi_0]}^{n-1}=\overbrace{[[Q_0,R],R\ldots,],R]}^{n}{}^{L}\mathcal{I}\,. \tag{4.39}$$

We can prove upper relation by mathematical induction. It was shown that this relation holds for $n=1,2$. Let us presume its validity for $n=N-1$. Then we have for $n=N$

$$\begin{aligned}\overbrace{[[[Q_0(\Phi_0),\Phi_0],\ldots],\Phi_0]}^{N-1} &= [\overbrace{[[Q_0,R],R,\ldots,]R]}^{N-1}{}^{L}\mathcal{I},\Phi_0] \\ &= \overbrace{[[Q_0,R],R],\ldots],R]}^{N-1}{}^{L}\mathcal{I}\star R^L\mathcal{I}-R^L\mathcal{I}\star\overbrace{[[Q_0,R],R],\ldots],R]}^{N-1}{}^{L}\mathcal{I} \\ &= \overbrace{[[Q_0,R],R],\ldots],R]}^{N-1}{}^{L}\,R\mathcal{I}-R\,\overbrace{[[Q_0,R],R],\ldots],R]}^{N-1}{}^{L}\mathcal{I} \\ &= \overbrace{[[Q_0,R],\ldots],R]}^{N}{}^{L}\mathcal{I}\end{aligned}$$

using results given above. We can then claim that $\Phi_0=R^L\mathcal{I}$ solves (4.22). We have also shown above that $A=e^{\Phi_0}Q_B(e^{-\Phi_0})$ solves the equation of motion (4.3). In other words, we have found classical field in the background independent NSFT that leads to the NSFT with correct BRST operator corresponding to some particular CFT background. It can be shown in the same way as in [28] that for some particular background CFT characterised by Q_B there is unique field $\Phi_0=R^L\mathcal{I}$.

In this section we have proposed background independent formulation (at least formally) of the NSFT. We have shown that any CFT background arises as a particular solution of the background independent NSFT theory. Of course, our calculation was pure formal as in case of [28] so that more detailed analysis should be done. We return to some open questions and suggestions for further work in conclusion.

5. Conclusion

In this short note we have proposed a background independent formulation of Berkovits' string field theory [4, 5, 6]. Our proposal is based on the form of the BRST operator for RNS string theory presented in [30]. Since the BRST operator we started in the background independent NSFT with contains ghost fields only, it is background independent as well. This is the situation similar to the case of vacuum string field theory [11, 17, 18, 21], where the BRST operator is constructed from the ghost field only and consequently VSFT is background independent. On the other hand, the BRST operator in VSFT has a vanishing cohomology, while in NSFT it has not. We mean that this is not in contradiction since our background independent NSFT theory corresponds to the BPS object so that there is no place for tachyon condensation. This remark also suggests possible limitation of our proposal. We mean that the true background independent formulation of supersymmetric string field theory will have such a form that will allow any solutions corresponding to BPS or non-BPS CFT background. In order to find such a formulation, we think that complete supersymmetric invariant action, including Ramond sector, should be found. We believe that such a formulation will be found in the near future and it will allow us to get new insight in the nature of supersymmetric theory.

We must also stress one important thing. Our calculation was pure formal in the sense that we have worked with Witten's star product as with an abstract object that does not depend on any background. It would be nice to perform more detailed analysis based on the CFT technique, in the similar way as in the beautiful paper [21]. We hope to return to this problem in the future.

Acknowledgments

We would like to thank N. Berkovits for useful comments and for pointing out the work [30]. This work was supported by the Czech Ministry of Education under Contract No. 1443100006.

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